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EFFICIENCY OF THE INVARIANT GRADIENT CRITERION OF STABILITY FOR THE FLIGHT CONDITIONS OF THE FLOW AROUND AXISYMMETRIC BODIES IN THE EARLY STAGE OF TRANSITION AND RELAMINARIZATION

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Quantitative results of the study of the invariant gradient criterion of stability, which was theoretically obtained by N. N. Yanenko and S. A. Gaponov, for the flight conditions of the flow around the forebodies of aerophysical complexes in the laminar flow regions and in the early stage of the laminar–turbulent transition and relaminarization are presented.

Introduction and Formulation of the Problems. To obtain reliable data on the thermal state of supersonic and hypersonic aircraft and their drag and dynamics, it is necessary to have credible results on the Reynolds numbers of the laminar–turbulent transition and relaminarization (reverse transition) in supersonic and hypersonic near-wall boundary layers. The problems of stability and transition to turbulence in incompressible and compressible laminar near-wall flows and relaminarization of turbulent boundary layers are rated among the most complicated problems in continuum mechanics. Their importance and practical interest are emphasized [1–5]. Despite the great number of theoretical papers, which are reviewed, for example, in [2, 4–6], there is no comprehensive theory of the transition of the laminar boundary-layer flow to a turbulent state. In connection with the use of effective numerical methods, techniques based on the theory of hydrodynamic stability, both linear [2, 4, 7, 8] and nonlinear [9–13], have been developed. Taking into account the difficulties of theoretical investigation of the transition in near-wall boundary layers and its practical importance, Yanenko and Gaponov [14] noted that it is important to determine the conditions of stability and transition on the basis of local flowfield properties.

It is impossible now to obtain reliable data on the transition in supersonic wind tunnels, first of all, because of the presence of an acoustic field in test sections [15, 16]. It is shown [17, 18] that the transition Reynolds numbers on the cones which were measured in flight were many-fold higher than those measured in wind tunnels. Along with the acoustic field, scaling effects make an additional contribution to the difference in ground-based and flight data on the laminar–turbulent transition for $M_\infty \geqslant 2.0$. These effects are caused by the fact that the full-scale Reynolds and Mach numbers and the temperature factor, the external flow perturbations, the effect of operating engines, and the aeroelastic properties of the bodies cannot be simultaneously simulated in supersonic and hypersonic wind tunnels. Reshotko [1] emphasized the necessity of careful preparation of flight experiments for studying the laminar–turbulent transition. The number of experiments conducted is rather small [17–21]. For example, a full-scale experiment was conducted at the Institute of Theoretical and Applied Mechanics (ITAM) of the Siberian Division of the Russian Academy of Sciences on the aerophysical complex "Oblako" ("Cloud") with a working engine [22]. The results obtained are in good agreement with the in-flight data of NASA [18] for free flying cones for identical Reynolds ($\mathrm{Re}_{1,\infty}$) and Mach (M_∞) numbers. However, the operating engines on the "Oblako" affected the transition for some Reynolds and Mach numbers.

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Thus, on the one hand, the final practical challenge of the theory of stability and transition is to predict the full-scale transition Reynolds numbers for bodies of various shapes. On the other hand, as is noted in [1], this is next to impossible to accomplish. It is also problematic to obtain reliable measurements of the transition Reynolds numbers close to the full-scale values using the models tested in supersonic wind tunnels [15, 16]. Reliable information on the laminar–turbulent transition in supersonic and hypersonic boundary layers has been obtained in flight experiments [18, 22].

The problem of relaminarization of a compressible turbulent boundary layer is also important in high-velocity aerodynamics. The effect of relaminarization was registered for the first time by A. I. Leontiev in 1952 in thermal measurements in the nozzle. This effect was experimentally verified later by Sternberg for a supersonic flow around a cylinder with a conical tip [23], in supersonic nozzles, in pipes, etc. [24–26], and in flight conditions on the forebody of the "Viking–10" rocket [20]. Nevertheless, the mechanism of relaminarization is poorly studied for hypersonic speeds. There are several papers in which relaminarization was studied theoretically [27, 28]. The scaling effects and the presence of acoustic fields in supersonic and hypersonic wind tunnels do not permit extrapolation of wind-tunnel data obtained in ground-based experiments to flight conditions.

In flight conditions, the effect of relaminarization of a supersonic turbulent boundary layer on a cone was registered on the "Viking–10" rocket by Snodgrass on the basis of thermal measurements [20]. For more complex flight conditions of a reusable and general-purpose aerophysical complex of the M100 type, the flight data of ITAM on relaminarization can be found in [29, 30]. Further flight experiments are necessary to study the relaminarization effect; otherwise, it is impossible to obtain reliable data on the drag, heat fluxes, and other characteristics of supersonic and hypersonic flying vehicles.

From the practical viewpoint, the problem of rapid determination of the flow regime on the basis of local properties of the medium is important [14, 31]. The Reynolds number Re is one of the most important criteria of similarity in modern aerodynamics. The traditional Reynolds number is also widely used in the theory of stability of laminar boundary layers. It does not remain invariant if the flow velocity changes by a constant value. In addition, the critical value of the Reynolds number differs for different types of flow by several orders of magnitude. Instability is related to flow inhomogeneity, and it is desirable to consider the gradients of the velocity rather than its absolute value.

The idea that the instability of laminar boundary layers is related to flow inhomogeneity, which is taken into account by the flow velocity gradient, was implemented earlier by van Driest and Blumer [32]. They obtained a gradient criterion of stability, which is based on the quantity

$$T_r = \frac{y^2}{\nu}\frac{du}{dy}.$$

Here y, u, and ν are the vertical coordinate, the velocity, and the kinematic viscosity in the boundary layer, respectively. Since this quantity contains the coordinate y, the corresponding criterion is not invariant relative to coordinate transformation.

A new invariant gradient criterion of stability of incompressible and compressible laminar near-wall boundary layers with account of local flow properties was developed [14, 31]. Yanenko and Gaponov [31] used the following linearized equations to study the stability of a plane-parallel flow to perturbations of the type $q = q(y)\exp[i(\alpha x + \beta z - \alpha c t)]$:

$$\begin{gathered} \rho i\alpha(u-c)f + \rho u'_y\varphi = -i\alpha\pi + \mu f''_{yy}, \quad \rho i\alpha(u-c)h = -i\beta\pi + \mu h''_{yy}, \\ \rho i\alpha(u-c)\varphi = -\pi'_y, \quad i\alpha(u-c)r + \rho'_y\varphi + (i\alpha f + i\beta h + \varphi'_y) = 0, \\ c_V\rho i\alpha(u-c)\vartheta + \rho c_V T'_y\varphi = -P(i\alpha f + i\beta h + \varphi'_y) + \lambda\vartheta''_{yy}, \quad \frac{\pi}{p} = \frac{r}{\rho} + \frac{\vartheta}{T}. \end{gathered} \tag{1}$$

Here u, ρ, T, and P are the main-flow velocity, density, temperature, and pressure, μ, λ, and c_V are the dynamic viscosity, thermal conductivity, and specific thermal conductivity at constant volume, f, h, φ, r, and ϑ are the amplitudes of perturbations of the longitudinal lateral and normal velocities, pressure, density, and temperature, and the subscript y denotes differentiation with respect to y.

The most intense fluctuations of the flow parameters (velocity, temperature, and density) are observed near the critical layer where the flow velocity u equals the phase velocity c. Thus, it can be assumed that the solution of system (1) is determined to a large extent by the local properties of the medium in the vicinity of this layer. Near the critical level, we can assume that $u - c = u'_y(y_c)(y - y_c)$, where y_c is the vertical coordinate of the critical layer; the remaining parameters of the main flow are equal to their local values.

It is shown in [31] that, making some substitutions and eliminating the pressure from Eq. (1), we can use a system of sixth-order differential equations to construct the solution in the vicinity of the critical layer. Applying the analysis of dimensionality, we can obtain the following invariant criterion of stability in the form of a gradient Reynolds number:

$$\mathrm{Re_{gr}} = \max_y \left(\frac{\rho u'_y}{\mu\alpha^2}\right)^{1/3},$$

where

$$\alpha = \max_{y=\mathrm{const}} \left\{ \left|\frac{u''_{yy}}{u'_y}\right|, \varepsilon_1 \left|u'_y \cos\left(\frac{\rho}{P}\right)^{1/2}\right|, \varepsilon_2 \left|\frac{\rho'_y}{\rho}\right|, \varepsilon_3 \left|\frac{a'_y}{a}\right| \right\}.$$

Here α is the wavenumber and a is the particle acceleration. The constants ε_i can be chosen on the basis of comparison of the data on stability. For example, Yanenko and Gaponov obtained $\varepsilon_3 \approx 1/36$ [31]. For compressible flows, ε_1 and ε_2 can be determined in a similar way.

Important factors are, first, the invariance of $\mathrm{Re_{gr}}$ and, second, its constancy or weak variation for Mach numbers $M_\infty \leqslant 4.0$. It should be noted that the gradient criterion was verified for comparatively simple flow conditions (velocity profiles from the Falkner–Skan family, a thermally insulated surface). The calculations [31] showed that for a laminar flow regime $\mathrm{Re_{gr}} \leqslant 30$. It is noted [14, 31] that the gradient Reynolds number $\mathrm{Re_{gr}}$ can also be used as a criterion of the laminar–turbulent transition.

The dominant role of flight experiments in obtaining reliable information on the supersonic laminar–turbulent transition and relaminarization is noted above. Taking into account the difficulties in conducting the flight experiments and interpreting the flight data, it is necessary to seek new ways for solving the problems described. One of the directions of their solution can be related to verification of the effectiveness and universality of the gradient number $\mathrm{Re_{gr}}$ in supersonic flight conditions. The physical and mathematical justification of the gradient criterion [31] and the possibility of obtaining a fast estimate of the loss of stability based on the data on the local flow properties seem to be important.

Thus, one problem is to obtain the quantitative data on the gradient number $\mathrm{Re_{gr}}$ [31] under full-scale flight conditions for operating solid-propellant rocket engines with account of gas-dynamic peculiarities of the flow for forebodies of the flying aerophysical rocket complexes

(1) "Oblako" in the early stage of the laminar–turbulent transition for $\mathrm{Re}_{L,\infty} \leqslant 2 \cdot 10^7$, $M_\infty \leqslant 2.0$, and acceleration $a \leqslant 12g$;

(2) M 100 in the early stage of relaminarization for $\mathrm{Re}_{L,\infty} \leqslant 10^8$, $M_\infty \leqslant 4.5$, and acceleration $a \leqslant 32g$.

Determination of the Gradient Number $\mathrm{Re_{gr}}$ for a Stable Laminar Supersonic Flow around the "Oblako" and M 100 Complexes. To solve various problems of high-velocity aerodynamics and thermal mechanics, including the problems of the laminar–turbulent transition, relaminarization, separation, interaction between separation and transition, aerodynamic heating, etc., a program of flight experiments was developed at the Institute of Theoretical and Applied Mechanics. Some issues of this program are described in [33]. The corresponding reusable and general-purpose aerophysical complexes are based on the M 100 and "Oblako" meteorological rockets. The results of 16 aerophysical flight experiments are presented in [29, 30, 34–36].

The gradient criterion for a supersonic flow around forebodies of the aerophysical M 100 and "Oblako" complexes is verified using the method of integration of numerical calculations and flight data, which was developed by A. M. Pavlyuchenko and realized [37, 38] using the technique in [39]. The idea of the method is to use flight data on the wall temperature of the M 100 or "Oblako" to numerically solve the system of equations of a compressible boundary layer. The problem in this formulation retains its correctness, and the reliability of the velocity (temperature) profiles calculated using real flight conditions increases. Since the derivatives u'_y and u''_{yy} are necessary to determine Re_{gr}, the velocity profiles were approximated by a smoothing cubic spline to avoid additional errors in calculating the derivatives.

Since the invariant criterion [31] was established from the analysis of the system of differential equations obtained from the general equations of stability for the critical layer, which is the region of maximum perturbations, it is of interest to study the dependence of the position of this layer over the height on the Mach number M_e at the outer edge of the laminar boundary layer. According to experimental data obtained by various authors and described by Henderson [17], the critical layer shifts from the depth of the boundary layer to its outer edge as the flow velocity increases from low to hypersonic values. The Mach numbers vary within $\mathrm{M}_e = 1.45$–1.65 at the beginning of the laminar–turbulent transition on the forebody of the "Oblako" flying complex [22] and within $\mathrm{M}_e = 3.3$–3.5 at the beginning of relaminarization on the M 100 complex [40]. For these values of M_e, the position of the critical layer, according to data [17], varies from $y_c/\delta \cong 0.13$ for the "Oblako" to $y_c/\delta \cong 0.38$ for the M 100 (y_c is the vertical coordinate of the critical layer and δ is the boundary-layer thickness). According to experimental results [3], the coordinate of the maximum perturbations does not exceed $y/\delta \cong 0.4$. Thus, for Mach numbers M_e realized under flight conditions on the "Oblako" and M 100 objects, the critical layer is located in the depth of the boundary layers considered, which allows one to correctly use the results for Re_{gr} [14, 31] for verification under flight conditions.

To solve these problems on the basis of the integration method, we used numerical integration of the system of equations of a compressible near-wall boundary layer, which was modified by Popkov [39]. It is shown [29, 30, 40] that, in an accelerated trajectory flight of the "Oblako" ($a \leqslant 12g$) and the M 100 ($a \leqslant 32g$, the gas-dynamic and thermal processes in the flow around the forebodies of these objects were practically stationary and the angle of attack in the flight with operating engines was close to zero. Since the boundary-layer thickness on these objects was much smaller than their diameter ($\delta \ll d$) and the flight was at almost zero incidence, and also the condition of quasistationary flow was fulfilled, the use of the system of equations of a stationary two-dimensional compressible boundary layer [39] was justified. This system was previously tested by comparing the numerical and experimental data for various aerodynamic characteristics.

The following initial system of averaged equations of a two-dimensional stationary boundary layer of a compressible gas is written using the commonly accepted notation:

$$u\,\frac{\partial u}{\partial x} + \nu\,\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dP}{dx} + \frac{1}{\rho}\frac{\partial}{\partial y}\Big[\mu\Big(1+\frac{\varepsilon}{\mu}\Big)\frac{\partial u}{\partial y}\Big]; \tag{2}$$

$$u\,\frac{\partial H}{\partial x} + \nu\,\frac{\partial H}{\partial y} = \frac{1}{\rho}\frac{1}{\mathrm{Pr}}\frac{\partial}{\partial y}\Big[\mu\Big(1+\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\frac{\varepsilon}{\mu}\Big)\frac{\partial H}{\partial y}\Big] - \frac{1}{\rho}\Big(\frac{1-\mathrm{Pr}}{\mathrm{Pr}}\Big)\frac{\partial}{\partial y}\Big[\mu\Big(1+\frac{1-\mathrm{Pr}_T}{1-\mathrm{Pr}}\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\frac{\varepsilon}{\mu}\Big)u\,\frac{\partial u}{\partial y}\Big]; \tag{3}$$

$$\frac{\partial(r_w\rho u)}{\partial x} + \frac{\partial(r_w\rho\nu)}{\partial y} = 0, \tag{4}$$

where P, Pr, Pr_T, μ, and ε are the pressure, the Prandtl number, the turbulent Prandtl number, and the molecular and turbulent viscosity, with the boundary conditions

$$\begin{gathered} y = u = 0, \quad \nu = \nu_w(x), \quad \Big(\bar\lambda_1 H + \bar\lambda_2\,\frac{\partial H}{\partial y}\Big)_{y=0} = \Delta(x), \\ y \to \infty, \quad u \to u_e(x), \quad H = H_0, \quad |\bar\lambda_1| + |\bar\lambda_2| \neq 0. \end{gathered} \tag{5}$$

The subscripts w, e, and 0 refer to the conditions at the wall, at the boundary-layer edge, and at the stagnation point, respectively, x and y are the longitudinal and normal coordinate on the body surface, u and ν and the longitudinal and normal components of the velocity vector, and H is the total enthalpy (the remaining variables are commonly used).

System (2)–(4) is integrated under the following assumption for the physical properties of the gas: the heat capacities C_p and C_V are constant. The following equation of state is valid:

$$P = \rho RT. \tag{6}$$

The dependence of molecular viscosity on temperature is determined by the Sutherland formula. The gas flow outside the boundary layer is isentropic and vortex-free.

In constructing the numerical algorithm, introducing new variables

$$\varphi = u/u_e, \qquad s = H/H_0 - 1; \tag{7}$$

$$\xi = \int_0^x \frac{\rho_e \mu_e}{\rho_0 \mu_0} \left(\frac{r_w}{r_0}\right)^2 (1 + \Gamma C_1 \mathrm{Re}_x^{C_2}) \frac{u_e}{u_\infty} \, dx; \tag{8}$$

$$z = \frac{\mu_e \rho_e}{\mu_0} \frac{u_e}{u_\infty} \frac{(1 + \Gamma C_1 \mathrm{Re}_x^{C_2})}{\sqrt{2\nu_0 \xi}} \frac{r_w}{r_0} \int_0^y \frac{dy}{\mu(1 + \varepsilon/\mu)}, \tag{9}$$

where

$$C_1 = 3.25 \cdot 10^{-3} \frac{\rho_w \mu_w}{\rho_e \mu_e} \left(\frac{T_r}{T_0}\right)^{0.4} (12.5 + 2.5\,\mathrm{M}_e + 0.5\,\mathrm{M}_e^2)^{-1}, \quad C_2 = 0.8,$$

and using the continuity equation (4), we can write the initial system (2), (3) as a system of two variables

$$\frac{\partial^2 \varphi}{\partial z^2} + \left(\Phi + \alpha \frac{\partial \Phi}{\partial \xi}\right) \frac{\partial \varphi}{\partial z} + \frac{\beta N_1}{1 - æ} (1 + s - \varphi^2) = \alpha N_1 \varphi \frac{\partial \varphi}{\partial \xi}, \tag{10}$$

$$\frac{\partial}{\partial z}\left(A_2 \frac{\partial s}{\partial z}\right) + \mathrm{Pr}\left(\Phi + \alpha \frac{\partial \Phi}{\partial \xi}\right) \frac{\partial s}{\partial z} - 2(1 - \mathrm{Pr}) æ \frac{\partial}{\partial z}\left(A_3 \varphi \frac{\partial \varphi}{\partial z}\right) = \mathrm{Pr}\, \alpha N_1 \varphi \frac{\partial s}{\partial z} \tag{11}$$

with the boundary conditions

$$z = 0: \quad \varphi = 0, \quad \Phi = \Phi_w(\xi), \quad |\lambda_1| s_w + |\lambda_2| \frac{\partial s}{\partial z}\Big|_{z=0} = \Delta(\xi), \quad z \to \infty, \quad \varphi \to 1, \quad s \to 0.$$

Here $N_1 = (\rho\mu/\rho_e\mu_e)(1 + \varepsilon/\mu)/(1 + \Gamma C_1 \mathrm{Re}_x^{C_2})$ is the reduced turbulent viscosity, $\Gamma = 0$ for the laminar regime, $\Gamma = 1$ for the turbulent regime, and

$$\Phi = \int_0^z N_1 \varphi \, dz, \quad \alpha = 2\xi, \quad \beta = \frac{\alpha}{u_e} \frac{du_e}{d\xi}; \quad æ = \frac{2u_e^2}{H_0}.$$

The coefficients A_2 and A_3 are the fractional-linear functions of the Prandtl numbers Pr, Pr_T, and turbulent viscosity. In calculations of the turbulent boundary layer, we closed system (2)–(4) by a modified model of the Prandtl mixing layer, which ensures good agreement of theoretical and experimental data for many practically important problems.

The algorithm and the difference scheme are described in detail in [39]. It should be noted that the use of turbulent viscosity in the transformation of the normal coordinate (9) substantially simplified the form of the equations, which remind one of the laminar boundary-layer equations and together with the normalization factor $1 + \Gamma C_1 \mathrm{Re}_x^{C_2}$ eliminate high gradients of the near-wall velocity and temperature profiles inherent in turbulent flows. Thus, the mathematical normal coordinate is automatically extended and the "stiffness" of the initial solutions is eliminated. Since the extension is always connected with the physical formulation of the problem and does not need any interference in most cases, a rather simple Crank–Nicholson difference scheme was used, which ensures the second-order accuracy for both coordinates. It is shown that the solution can be obtained on one grid for all flow regimes in the boundary layer from laminar ($\Gamma = 0$) to turbulent ($\Gamma = 1$) in a wide range of Mach and Reynolds numbers, suction and injection parameters, pressure gradients, roughness, and heat transfer. A constant integration step can be used across the boundary layer, which significantly simplifies the numerical realization of the algorithm and makes it universal for a large class of problems. The code for a two-dimensional stationary laminar and turbulent boundary layer was comprehensively tested and accepted for the state foundation of algorithms and codes as part of the packet of codes. The following studies were conducted for flight testing of the gradient number $\mathrm{Re}_{\mathrm{gr}}$.

1. The method of integration of numerical calculations and flight data on the wall temperature, which are used in the boundary conditions of the system of equations of a compressible boundary layer, was tested.

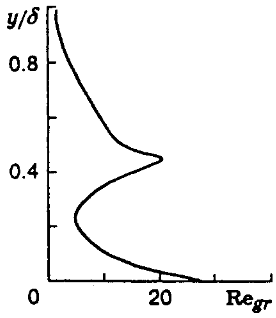

Fig. 1

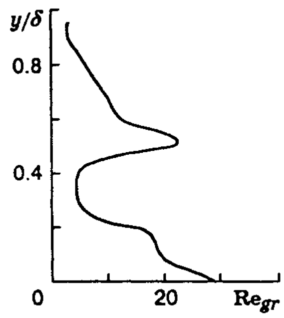

Fig. 2

The results of flight experiments for the wall temperature of the forebodies of the "Oblako" and M 100 axisymmetric complexes with an attached flow are presented [34]. The measurement error was less than 2% for the M 100 and 0.5–1% for the "Oblako" [30, 34]. Verification of the integration method consisted of determining the friction drag of the forebody of the M 100 for the case of a turbulent attached flow with heat transfer and comparing the experimental and calculated results. Possessing the velocity profiles along the forebody of the M 100 for different Mach numbers, which were calculated by the above method, using a smoothing cubic spline, and correctly determining the velocity derivative u'_y on the wall, we can calculate the friction drag at the examined points along the body on the basis of the Newton law of friction. A comparison of the friction drag results obtained for the M 100 with the wind-tunnel data obtained by Moore, Harkness, and other authors [41] with the universal integral curve of Wilson and the calculations based on the theory proposed by Kutateladze and Leontiev [42] demonstrated the effectiveness of the method of integration of numerical and flight data. The corresponding results of this comparison are presented [30, 37, 38].

2. The theoretical result obtained by Yanenko and Gaponov [31] that $\mathrm{Re}_{\mathrm{gr}} \leqslant 30$ for stable laminar flows was tested under flight conditions in the regions of supersonic laminar flow around the forebodies of the aerophysical complexes "Oblako" and M 100. The boundary-layer equations (10) and (11) for the laminar flow regime were integrated.

Using a direct comparison of calculations and flight experiments on the wall temperature and the stability theory, it is shown [22, 29] that the laminar and transitional flow regimes in the boundary layer are formed on the forebody of the "Oblako" complex. By comparing the numerical and flight data on the wall temperature, it is found [29, 40] that a turbulent flow regime is formed on the forebody of the M 100 complex after take-off and a quasilaminar flow regime is formed as a result of the relaminarization effect as the M 100 moves along the trajectory. The presence of stable laminar and quasilaminar supersonic flow regimes with operating engines on the forebodies of the "Oblako" and M 100 objects allowed one to obtain the distribution of $\mathrm{Re}_{\mathrm{gr}}$ over the height of laminar boundary layers and verify the theoretical estimates [31] under the conditions of aerodynamic heating.

Figures 1 and 2 show the distributions of $\mathrm{Re}_{\mathrm{gr}}$ over the thickness of supersonic laminar boundary layers for the "Oblako" meteorological rocket and the M 100 under the conditions of aerodynamic heating and operating engines, which were discussed earlier [37, 38]. It is seen than $\mathrm{Re}_{\mathrm{gr}} < 30$ over the height of stable laminar boundary layers (this does not contradict the theoretical estimates [31]), and there are two maximum values $\mathrm{Re}_{\mathrm{gr}}^{\mathrm{max}}$ over the thickness of the boundary layers (inside the layer and near the wall), which indicate the possibility of the loss of stability both inside the boundary layer and near the wall under nonstationary actions. According to [31], the loss of stability of a laminar boundary layer occurs if $\mathrm{Re}_{\mathrm{gr}}^{\mathrm{max}}(y) > 30$.

The data on the distribution $\mathrm{Re}_{\mathrm{gr}}(y)$ for the flight conditions of the aerophysical complexes "Oblako" and M 100 (Figs. 1 and 2) allowed us to pass to solving the problems of verification of these criteria in the early stage of the laminar–turbulent transition on the "Oblako" complex and relaminarization on M 100.

Distribution of the Gradient Number Re_{gr} in the Regions of the Beginning of Transition on the Flying Complex "Oblako" and at the Beginning of Relaminarization on the M 100 Complex. We proved above the effectiveness of the procedure of numerical calculation of Re_{gr} for the regions of stable laminar flow around the forebodies of the "Oblako" complex (in the region before the transition point) and the M 100 (in the region after the beginning of relaminarization). This result can be considered as a basis for solving two more complicated problems:

(1) determination of the distributions $Re_{gr}(y)$ over the height of a supersonic boundary layer at the beginning of the laminar-turbulent transition on the forebody of the "Oblako" complex;

(2) determination of the distributions $Re_{gr}(y)$ in a supersonic boundary layer at the beginning of relaminarization on the forebody of the M 100.

These problems are considered under the conditions of operating solid-propellant rocket engines of the flying complexes and simultaneous variation of the Reynolds and Mach numbers and the temperature factor in the trajectory flight. The results of flight experiments for the transition Reynolds numbers on the "Oblako" complex and relaminarization on the M 100 are presented in [22, 29]. Each particular time of the beginning of transition on the "Oblako" complex or relaminarization on the M 100 at different points of their forebodies corresponded to a combination of flight values of the Reynolds and Mach numbers and the temperature factor. The laminar-turbulent transition at three points on the forebody of the "Oblako" complex occurred at τ = 6–7 sec, and the relaminarization at 12 points on the forebody of the M 100 began at τ = 14–16 sec [22, 29, 40].

Based on the method of integration of numerical methods and flight values of the wall temperature used in the boundary conditions, the velocity profiles were calculated for laminar flow regimes from the tip of the forebodies of the flying complexes to the points at which the transition ("Oblako") or relaminarization (M 100) occurs at a fixed time τ, which was physically justified [29, 36, 37, 40]. The velocity profiles along the forebodies of the "Oblako" and M 100 were determined at the points of the beginning of transition on the "Oblako" and relaminarization on the M 100 using the flight data on the wall temperature of these objects in the boundary conditions of system (2)–(11). After that, the distributions of Re_{gr} were found using smoothing cubic splines for approximation of the calculated velocity profiles.

Figure 3 shows the distribution $Re_{gr}(y/\delta)$ at the point of the beginning of the laminar-turbulent transition on the forebody of the aerophysical complex "Oblako" in three cross sections over the length: curve 1 for x = 0.4 m and M_e = 1.45 (the wall material is 1Kh18N9T and its thickness is δ_w = 1 mm), curve 2 for x = 0.25 m and M_e = 1.56 (the wall material is D16T and its thickness is δ_w = 4 mm), and curve 3 for x = 0.28 m and M_e = 1.65 (the wall material is D16T and its thickness is δ_w = 1.8 mm). Each curve corresponds to one of three combinations of Re_x, M_e, and $\bar{T}_w$. The x coordinate is laid off from the tip of the forebody along the longitudinal axis, M_e is the Mach number at the outer edge of a compressible boundary layer, and $\bar{T}_w$ is the temperature factor.

The distributions $Re_{gr}(y/\delta)$ (curves 1–3) at the points of the beginning of the laminar-turbulent transition on the "Oblako" have one maximum near the wall, in contrast to two maxima in the distributions $Re_{gr}(y/\delta)$ in the region of a stable laminar flow around this object (see Fig. 1). The data in Fig. 1 indicate a possible loss of stability near the wall or inside the boundary layer, whereas Fig. 3 shows that the laminar-turbulent transition is caused by the development of perturbations near the wall at $y/\delta \cong$ 0.075–0.2. The values of $Re^{max}_{gr,1,2,3}$ = 28–49 at three transition points on the "Oblako" (Fig. 3) are greater than Re^{max}_{gr} = 26.5 (see Fig. 1) for a stable laminar boundary layer on the "Oblako," which is physically correct. Some data on Re_{gr} at the beginning of transition at one point on the forebody of the "Oblako" complex are presented in [38].

The quantitative results on the distribution $Re_{gr}(y/\delta)$ at the beginning of relaminarization of a supersonic turbulent boundary layer and, as is shown by Pavlyuchenko [29] and Maksimova and Pavlyuchenko [40], at the beginning of a practically laminar flow regime on a steel forebody of the aerophysical complex M 100 are plotted in Fig. 4: curve 1 for x = 0.49 m and M_e = 3.5, curve 2 for x = 0.245 m and M_e = 3.4, and curve 3 for x = 0.139 m and M_e = 3.3. Each curve corresponds to one of the combinations of Re_x, M_e, and the temperature factor. The flight values of the wall temperature in time and over the length of the forebody

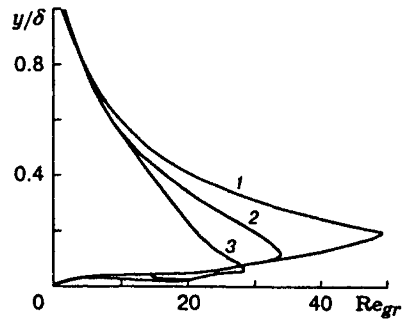

Fig. 3

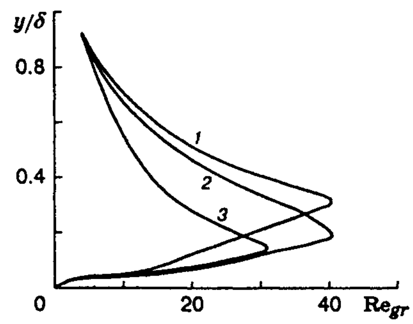

Fig. 4

of the M 100 that are used in the boundary conditions of system (2)–(11) are presented in [34].

It follows from Fig. 4 that the distributions $\mathrm{Re_{gr}}(y/\delta)$ (curves 1–3) at the points of the beginning of relaminarization on the forebody of the M 100 have one maximum near the wall, in contrast to two maxima in the distributions $\mathrm{Re_{gr}}(y/\delta)$ in the region of a stable laminar flow around this object (see Fig. 2). The distributions $\mathrm{Re_{gr}}(y/\delta)$ in Fig. 2 indicate a possible suppression of turbulence near the wall or inside the boundary layer, whereas the results in Fig. 4 allow us to conclude that the degeneration of the turbulent flow regime is caused by the factors acting near the wall at $y/\delta \cong 0.14$–0.32. The difference in the curves in Fig. 4 is explained by some difference in the flow and heat-transfer conditions for three combinations of the Mach and Reynolds numbers and the temperature factor. The values $\mathrm{Re}^{\max}_{\mathrm{gr},1,2,3} = 30.6$–$40$ exceed $\mathrm{Re}^{\max}_{\mathrm{gr}} = 28.5$ (see Fig. 2) for a stable, almost laminar boundary layer formed on the forebody of the M 100 as a result of relaminarization. The previously obtained data on $\mathrm{Re_{gr}}$ at the beginning of relaminarization at one point on the forebody of the M 100 are presented in [38].

Conclusions. (1) The gradient criteria of stability, which were obtained theoretically by Yanenko and Gaponov [14, 31] and take into account the local flow properties, have been tested for the first time for the conditions of supersonic flight of axisymmetric aerophysical complexes M 100 ($\mathrm{Re}_{L,\infty} \leqslant 10^8$, $\mathrm{M}_\infty \leqslant 4.5$, and $a \leqslant 32g$) and "Oblako" ($\mathrm{Re}_{L,\infty} \leqslant 2 \cdot 10^7$, $\mathrm{M}_\infty \leqslant 2.0$, and $a \leqslant 12g$) at zero incidence and operating solid-propellant rocket engines. The concept of integration of the numerical method and the flight values of the temperature of the object surfaces was used, which ensured the reliability of the results obtained.

(2) The mathematically justified invariant number $\mathrm{Re_{gr}}$ varies in rather narrow limits ($\mathrm{Re}^{\max}_{\mathrm{gr}} = 26.5$–$49$) under the flight conditions for fundamentally different flow regimes in a supersonic boundary layer on the forebodies of the aerophysical complexes M 100 and "Oblako" (the laminar regime, the beginning of the laminar-turbulent transition, the beginning of relaminarization), which proves the universal character of this parameter.

(3) The results obtained on the distribution of $\mathrm{Re_{gr}}$ (Figs. 1–4) extend the theoretical concepts of Yanenko and Gaponov to the flight conditions of the flow around the "Oblako" and M 100 objects under the conditions of compressibility, aerodynamic heating, operating engines, acceleration, laminar–turbulent transition, and relaminarization.

(4) For the flow around the forebodies of the flying axisymmetric complexes "Oblako" and M 100, it has been established that the gradient criterion allows one to determine the stability or instability of the laminar flow regime under flight conditions. This criterion can be used as a criterion of the laminar–turbulent transition (see Fig. 3) and relaminarization (see Fig. 4) for the examined range of parameters under the flight conditions of the two-dimensional flow. The invariant criterion has a certain universality. Further studies of this criterion under the flight conditions of the flow in a wide range of variation of $\mathrm{Re}_{L,\infty}$, M_∞, and $\bar{T}_w$ are necessary.

(5) The data on the weak dependence of the critical value of $\mathrm{Re_{gr}}$ on the flow parameters offer new possibilities for studying the stability, transition, and relaminarization under the flight conditions of supersonic flow around bodies, which cannot be simulated in wind tunnels, and contribute to solving the problem of

scaling effects in high-speed aerodynamics, which are caused by the known limitations of the currently existing supersonic wind tunnels and theoretical methods.

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